Analytical time-dependent functions describing the change of the concentration of the solvent $S(t)$ and the homeopathic active substance $A(t)$ during decimal and centesimal dilution are derived. The function $S(t)$ is a special case of the West–Brown–Enquist curve describing ontogenic growth, the increase in concentration of the solvent during potentization resembles the growth of biological systems. It is demonstrated that the macroscopic $S(t)$ function is the ground state solution of the microscopic non-local Horodecki–Feinberg equation for the time-dependent Hulthén potential at the critical screening. In consequence potentization belongs to the class of quasi-quantum phenomena playing an important role both in biological systems and homeopathy. A comparison of the results predicted by the model proposed with the results of experiments on delayed luminescence of a homeopathic medicine is made. *Homeopathy* (2011) 100, 259–263.

**Keywords:** Potentization; Succussion; Vital force; Ontogenic growth; Hulthén potential; Non-locality

**Introduction**

Homeopathic medicines are derived from botanical, animal or mineral sources by a successive dilution and vigorously shaking (succussion) referred to as potentization. This process converts the original substance into a therapeutically active medicine of decimal (1:10), centesimal (1:100) or quinquaginta millesimal (Q- or LM-potency 1:50 000) dilution scales. The homogenization of active substance involves, for liquids, shaking; insoluble substances are homogenized by grinding (trituration). The dilution attained after one decimal dilution is termed $D_1$ and is used as the starting point for preparing the next dilution, $D_2$ in exactly the same manner. This process can be continued indefinitely even beyond Avogadro number, when no molecules of the active substance are present in medicine. To explain activity at high dilutions researchers have used theoretical models which refer to water polymers, clathrates, electric dipoles, vortices and other mechanisms and structures assumed to be carriers of information transferred from the molecules of the active substance to the ordered molecules of the solvent produced by potentization. According to the models specified this information is administered during a homeopathic treatment.

A careful reading of the Hahnemann’s *Organon of Medicine* reveals that he believed in the possibility of exciting in the homeopathic medicine a spirit-like power of medicines (geistartige Kraft der Arzneien) or a vital force by potentization. According to Hahnemann this vital force is immanent component of the homeopathic medicine besides the solvent (water, ethanol, lactose) and active substance employed in its production. It is interesting that the concept of spirit-like power of homeopathic drugs has been abandoned in contemporary homeopathy. The main objective of the present work is to demonstrate that the concentration of the solvent in which the active homeopathic substance is diluted increases according to the function which is a special case of the West–Brown–Enquist curve describing ontogenic growth. Hence, the medicine prepared according to the homeopathic method should be endowed with a power to growth — the same as a growing biological system. I will compare the model proposed with recently performed experiment on delayed luminescence.

**Mathematics of potentization**

Let’s assume that the active substance of mass $m_A$ is dissolved and homogenized in the solvent of mass $m_S$ by making use of the succussion procedure and decimal dilution. In such circumstances at every step of potentization the following relationship is satisfied:
\[ m_A + m_S = M \] (1)

In which \( M = \text{const} \) is the total mass of the medicine prepared. For example, \( D_1 \) and \( D_2 \) potencies can be described by the formulae (the unit of mass is grams)

\[
D_1 : \frac{1[g] + 9[g]}{10} = 0.1[g] + 0.9[g] = 1[g]
\]

\[
D_2 : \frac{1[g] + 9[g]}{10} = 0.01[g] + 0.99[g] = 1[g]
\]

In the similar manner one may produce further potencies using both decimal and centesimal dilution. The results are presented in Table 1.

Dividing the mass relation (1) by \( M \) one gets

\[
A(x) + S(x) = \frac{m_A}{M} + \frac{m_S}{M} = 1
\] (2)

In which \( A(x) = m_A/M \) and \( S(x) = m_S/M \) multiplied by 100 denote the concentration of the active substance \( A(x) \) and solvent \( S(x) \) in medicine, \( x = 1, 2, 3, \ldots N \) stands for the step of the potentization. The functions \( A(x) \) and \( S(x) \) can also be interpreted as the probability of finding a molecule of the active substance or a molecule of the solvent in the homeopathic medicine. The results presented in Table 1 show that for the decimal dilution \( A(x) \) and \( S(x) \) can be specified as

\[
A(x) = 10^{-x} \quad S(x) = 1 - 10^{-x}
\] (3)

for centesimal dilutions we have

\[
A(x) = 100^{-x} = 10^{-2x} \quad S(x) = 1 - 100^{-x} = 1 - 10^{-2x}
\] (4)

### Potentization time

According to Hahnemann\(^9\), the potentization (dynamization) of the medicine is obtained by a sequence of succussions and dilutions in a given time sequences, or by an exact number of mixing (triturations) of a diluted medicinal substance. For example, the dispersion and homogenization of the active substance and the solvent in the homeopathic medicine. Applying the same mathematical procedure for centesimal dilutions one gets the formulae

\[
a = \frac{\ln(10)}{t_0} = \frac{2.302585093}{t_0}
\] (7)

The functions \( A(t) \) and \( S(t) \) determine the concentration of the active substance and the solvent in the medicine at each step of the potentization procedure performed at the time \( t_0 \) including succussion and dilution. They accurately reproduce the experimental data points presented in Table 1 for \( t = nt_0, n = 1, 2, \ldots \) whereas the periods \( <0, t_0>, <t_0, 2t_0> \) etc. are only approximations to the true curves describing the concentration of the active substance.

### First- and second-order dynamization

The function \( S(t) \) describing the increase in the solvent concentration during potentization satisfies the first- and second-order differential equations

\[
\frac{d}{dt} S(t) - a \frac{\exp(-at)}{1 - \exp(-at)} S(t) = 0
\] (8)

\[
\frac{d^2}{dt^2} S(t) + a^2 \frac{\exp(-at)}{1 - \exp(-at)} S(t) = 0
\] (9)

The second term in the above equations represents the well-known in the quantum physics Hulthén potential\(^12\) widely used in the description of electrostatic interactions between micro-particles. The equation (9) can be expressed in the dimensionless coordinate \( \tau = at \)

\[
\frac{d^2}{d\tau^2} S(\tau) + \frac{\exp(-\tau)}{1 - \exp(-\tau)} S(\tau) = 0
\] (10)

One may prove that the above equation is a special case of the quantal non-local Horodecki–Feinberg equation\(^10,11\) for the time-dependent Hulthén potential\(^12\) at the critical screening\(^13\) (see Technical appendix). This result indicates that the process of increasing concentration of the solvent during preparation of the homeopathic...
Ontogenic growth and potentization

The function $S(\tau)$, which describes the concentration or the solvent in the medicine at each step of the potentization, is well known in the biological domain. In 2001 West, Brown and Enquist\textsuperscript{15} (WBE) formulated a general model for ontogenic growth from the first principles. On the basis of the conservation of metabolic energy, the allometric scaling of metabolic rate, and energetic costs of producing and maintaining biomass, they derived the function

$$m(t) = M[1 - c_0 \exp(-c_1 t)]^{\frac{1}{2}}$$  \hspace{1cm} (11)

which fits very well the data for a variety of different species from protozoa to mammalians. Here, $m_0$ is the initial mass of the system, $M$ denotes the maximum body size reached whereas $a_0$ is the metabolic parameter.\textsuperscript{15} The function (11) can be expressed in dimensionless form

$$r(\tau) = 1 - \exp(-\tau) \quad r(\tau) = (\frac{m_0}{M})^{\frac{1}{2}}$$ \hspace{1cm} (13)

in which

$$\tau = c_1 (t - t_e) \quad t_e = \ln(c_0)/c_1$$ \hspace{1cm} (14)

As WBE\textsuperscript{15} showed, equation (13) provides a powerful way of plotting the data that reveals universal properties of biological growth. If the mass ratio $r(\tau)$ is plotted against a variable $\tau$ then all species (mammals, birds, fish, crustacea), regardless of taxon, cellular metabolic rate and mature body size $M$ fall on the same parameterless universal curve $r(\tau)$.

Equation (13) reveals that function $r(\tau)$ has identical form as homeopathic function $S(\tau)$. A detailed analysis of this mathematical similarity leads to the conclusion that the homeopathic function $S(t) = m(t)/M$ describing the relative change of mass of the solvent during dilution is a special case of the time-dependent WBE function (11)

$$m(t) = M[1 - c_0 \exp(-c_1 t)]^{\frac{1}{2}} \quad m(t) = M[1 - \exp(-at)]$$ \hspace{1cm} (15)

A comparison of expressions (15) reveals that the latter can be obtained from the former by the substitutions

$$c_0 \rightarrow 1 \quad c_1 \rightarrow a \quad c_2 \rightarrow 1$$ \hspace{1cm} (16)

The condition $c_0 = 1$ reflects the fact that the initial mass of the solvent during potentization is equal to zero, $c_2 = 1$ indicates the different mass scaling of this process in comparison with the mass scaling of the biological growth ($c_2 = 1/4$). We conclude that the concentration of the solvent in which the active homeopathic substance is diluted increases according to the function $m(t)$, which is, a special case of the West–Brown–Enquist curve describing ontogenic growth. Hence, the medicine prepared according to the homeopathic method should be endowed with a growth (vital) force or using Winsor’s\textsuperscript{16} notion power to growth, the same as a growing biological system. The growth force associated with the Hulthén potential appearing in (10) one may calculate in the same manner as the electric force is calculated in electrodynamics: it is a negative temporal derivative of the vector potential\textsuperscript{17,18} appearing in Eq. (A1).

In the dimensionless temporal coordinate the growth (vital) force associated with living and homeopathic systems is represented by the same formula

$$F(\tau) = -\frac{d}{d\tau} \left[ \frac{\exp(-\tau)}{1 - \exp(-\tau)} \right] = -\frac{\exp(-\tau)}{[1 - \exp(-\tau)]^2}$$ \hspace{1cm} (17)

Conclusions

From the scientific point of view potentization seems to be an irrational and mysterious procedure, difficult to explain by well-established physical theories. The results of this work indicate that potentization can be considered in rational terms using the concepts of potentization time and molecular dispersion. When the active substance is diluted in the solvent and then vigorously shaken with striking against an elastic stop (succussion) two processes take place on the micro-level: first — molecular dispersion of the substance, and second — removal of the active molecules of the starting substance.

From the physical point of view, the precise numbers of shakings and dilutions in a given time sequences, or by an exact number of triturations of a diluted medicinal substance is a kind of clock, which permits introduction of the potentization time and description of the time-change of the concentration of the solvent in the medicine by the function $S(t)$. This function is a special case of the universal WBE function describing the ontogenic growth. If biological growth according to function (13) is characterized by a power to grow (17) then the potentization procedure should excite the same force in the solvent during preparation of the homeopathic medicine. In other words, the increase in concentration of the molecules of the solvent during potentization resembles growth of biological systems. This conclusion gives rise to an hypothesis, consistent with Hahnemann\textsuperscript{9}:

...remarkable transformation of the properties of natural bodies through the mechanical action of trituration and succussion on their particles (while these particles are diffused in an inert dry or liquid substance) develops the latent dynamic powers previously imperceptible and as it were
lying hidden asleep in them. These powers electively affect
the vital principle of animal life. This process is called
dynamization or potentization (development of medicinal
power), and it creates what we call dynamizations or poten-
cies of different degrees.

The second aspect of potentization — molecular disper-
sion — is a condition sine qua non for the total homogeni-
zeation of the active substance in the solvent as only then are
the functions \( A(t) \) and \( S(t) \) satisfied; \( S(\tau) \) has identical form
as WBE function describing the ontogenic growth. In such
circumstances the medicine should be endowed with a vital
force — the same as a growing biological system. In this
picture the process of preparation of the homeopathic med-
cine reproduces the biological growth and excites in the
medicine a spirit-like power of medicines.

The removal of the active molecules of the tincture in the
series of dilutions results in diminishing the mean distance
between molecules of the solvent and increasing interac-
tions between them. This effect is connected with increasing
value of the Hulthén potential energy (see Figure 1) of
the solvent justifying the homeopathic terms: potentization
and dynamization of the homeopathic medicine during suc-
cussion and dilution.

Because the macroscopic function \( S(\tau) \) describing the
change in concentration of the solvent in the medicine is
a special case solution of the microscopic non-local quantal
Horodecki—Feinberg equation (A1), the potentization be-
longs to the class of phenomena describing by the non-
local quasi-quantum Eq.(10). The non-local character of
this process causes that the succussion generates molecules
of the solvent in the correlated (quasi-entangled) state ame-
able to form complex structures against decoherence due
to collisions with other molecules, exchanging the electro-
magnetic radiation and chaotic thermal influences. The re-
results reported by Del Guidice et al
\(^3\) confirmed that the
water molecules can move in highly correlated and ordered
way due to interactions between water electric dipole and
radiation field, which produce quasi-ordered structures in
macroscopic domain. According to Weingärtner,\(^8\) such
correlated molecular configurations can be effective car-
rriers of information between molecules of the active sub-
stance and molecules of the solvent, during preparation
of homeopathic medicines. Hence, they can play the role
of material carriers of information that is administered dur-
ing a homeopathic treatment. This interpretation admits
homeopathic activity even if no molecules of the active
substance are present in medicine. It is consistent with
the Collins\(^19\) model assuming that when the active sub-
stance dissolved in water becomes more dilute, the remain-
ing molecules clump together to form aggregates of
increasing size. Such aggregates endowed with a vital force
could affect biological systems, hence providing some pos-
sible explanation for the effect of a homeopathic activity.

The theoretical results obtained in this work are consist-
ent with the results of the Lenger—Bajpai—Drexel\(^20\)
experiments on delayed luminescence on Argentum metallicum. Delayed luminescence is the phenomenon of
photon emission by a complex living system after exposure
to light for a few seconds. The photon is observed after
a few milliseconds delay and is observable for a few min-
utes. The shape of the signal can be analyzed in terms of
four parameters: \( t_0, B_0, B_1, B_2 \) describing the change in
time of the numbers of photons emitted

\[
n(t) = B_0 + B_1/(t + t_0) + B_2/(t + t_0)^2 \tag{18}
\]

The coefficients \( B_0 \) and \( B_1 \) take significant values in liv-
ing systems while coefficient \( B_2 \) makes a contribution in
non-living complex systems. The delayed luminescence
signals of Argentum metallicum were characterized by
the coefficient \( B_2 \) typical of the delayed luminescence of
non-living systems, but also by the coefficient \( B_0 \) typical
of living systems. Both coefficients indicate the presence
of holistic quantum structures in homeopathic medicine\(^20\)
and suggest characteristics similar to those observed in liv-
ing systems.

**Technical appendix**

The non-local quantum states of a particle of mass \( m \)
moving with superluminal velocity in the field of the time-
dependent vector potential \( V(t) \) is described by the Horo-
decki—Feinberg equation\(^10,11\)

\[
\frac{\hbar^2}{2mc^3} \frac{d^2\Psi}{dt^2} + \frac{1}{c} V(t)\Psi = \Psi \tag{A1}
\]

Here \( \Psi \) represents a non-local matter wave associated
with the superluminal particle of momentum \( P \), \( \hbar = 1.05457266 \times 10^{-34} \) Js is the Planck constant divided
by \( 2\pi \), \( c \) is the light velocity. Equation (A1) represents
the non-relativistic version of the relativistic Feinberg
equation\(^10\) for non-local faster than light objects. It has
been derived by Horodecki\(^11\) by taking advantage the
same procedure as that used in deriving the Schrödinger
equation from the relativistic Klein-Gordon equation for
local slower than light particles.

The Horodecki—Feinberg equation with the time-
dependent Hulthén potential\(^12\)
\[
\frac{\hbar^2}{2mc^2} \frac{d^2}{dt^2} \psi_v - \frac{V_0}{1 - \exp(-c_1(t - t_e))} \psi_v = P_1 \psi_v.
\]

(A2)

can be transformed to a dimensionless form\(^\text{21}\)
\[
\frac{d^2}{dt^2} \psi_v + \beta^2 \frac{\exp(-\tau)}{1 - \exp(-\tau)} \psi_v = \epsilon^2 \psi_v, \quad \epsilon = \frac{\beta^2 - v^2}{2\beta^v},
\]

\(v = 1, 2, 3\ldots\)

(A3)
in which
\[
\tau = c_1(t - t_e), \quad \beta^2 = \frac{2mc^2V_0}{\hbar^2 c_1^2}, \quad \epsilon^2 = \frac{2mc^2P_1}{\hbar^2 c_1^2}.
\]

(A4)
The eigen functions of the equation (A3) take the form\(^\text{21}\)
\[
\psi_v = [1 - \exp(-\tau)] \left[ \exp(-\epsilon \tau) \right] F_1[2\epsilon + 1, v, 1 - \nu, 2\epsilon_v + 1; \exp(-\tau)]
\]

(A5)
in which \(F_1\) denotes hypergeometric function. For \(\beta = 1\) and ground state \(v = 1\) we have \(\epsilon_1 = 0\) and \(P_1 = 0\), whereas the ground state solution \(\Psi_1\) reduces to the function \(S(\tau)\) describing the concentration of the solvent in the homeopathic medicine
\[
\Psi_1 = [1 - \exp(-\tau)] \left[ \exp(-\epsilon_1 \tau) \right]^{\nu=1,\nu=1} S(\tau) = 1 - \exp(-\tau)
\]

(A6)

This result indicates that the universal WBE growth function \(r(\tau)\) and homeopathic function \(S(\tau)\) can be identified with the ground state solution of the Horodecki—Feinberg equation for the time-dependent Hultén oscillator at the critical screening.\(^\text{15}\) Then \(\beta = 1\) and the momentum eigen value is equal to zero \(P_1 = 0\). In such circumstances both the biological growth and the change in concentration of the solvent during potentization of the homeopathic medicine belong to the class of quasi-quantum phenomena.\(^\text{22,23}\)

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