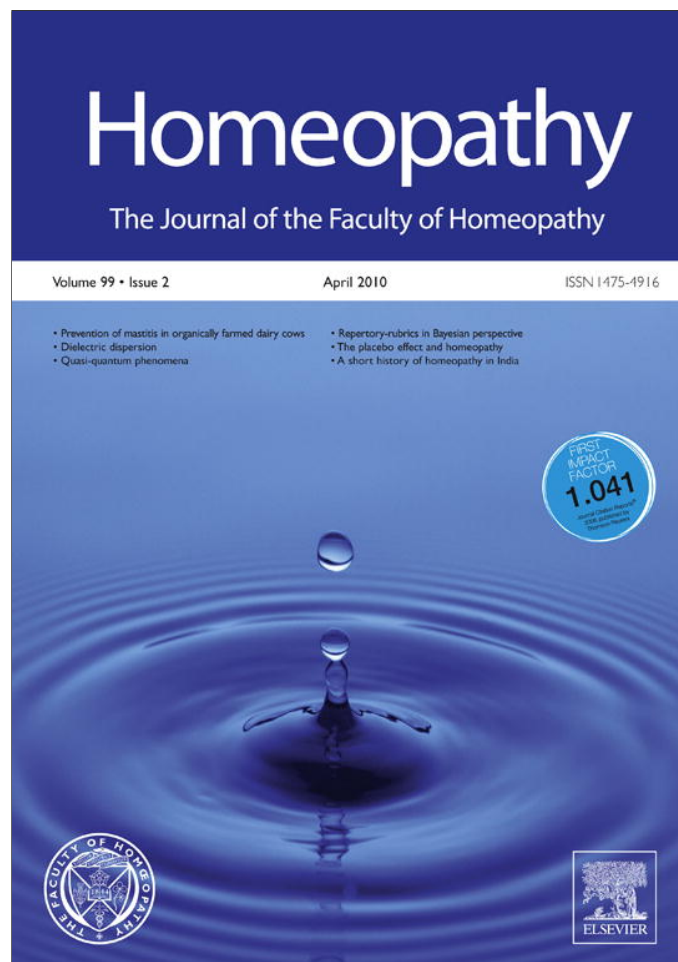


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ORIGINAL PAPER

Quasi-quantum phenomena: the key to understanding homeopathy

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On the basis of the first- and second-order Gompertzian kinetics it has been proved that the crystallization and its reciprocal process of dissolution belong to the class of quasi-quantum non-local coherent phenomena. Hence, there exists a direct link to homeopathy: molecules of the remedy prepared in the process of dilution of the active substance are non-locally interconnected at-a-distance. The results obtained provide strong arguments justifying formulated *ad hoc* macroscopic versions of quantum non-locality, entanglement and coherence employed in interpretation of the homeopathic remedies activity and effectiveness. In particular they are consistent with the predictions of the weak quantum theory developed by Atmanspacher and coworkers. *Homeopathy* (2010) 99, 104–112.

Keywords: Gompertzian systems; Zwietering–Gompertz function; Quantum coherence; Non-locality; Entanglement; Crystallization and dissolution; Homeopathy

Introduction

In order to explain theoretically the enigma of homeopathy, researchers employ either well established physical concepts like thermodynamics,¹ chaos,² field,³ quantum theory^{4,5} or generalize existing models to obtain e.g. *weak* version of the quantum mechanics⁶ or macroscopic version of quantum entanglement^{7–9} and non-locality.¹⁰ The second approach seems to be more prosperous, although controversial (mainly due to the decoherence problem), as the orthodox quantum theory is difficult to apply on the macroscopic scale, which is a genuine scene for homeopathic processes, i.e. remedy preparation and therapy. It is generally assumed that typical microscopic effects as quantization, superposition, correlation or uncertainty do not appear in macroscopic world. There exist, however, the macroscopic quantum phenomena of the first kind (superconductivity, superfluidity) and the second kind (macroscopic quantum tunnelling *via* Josephson junction), but they play significant role at low temperatures when there is little thermal motion present to mask the quantum nature of a substance.^{11,12} Hence, the macroscopic

quantum phenomena of the first and second kind cannot be employed in investigation of homeopathic remedies activity and their effectiveness. On the other hand, researchers discovered that some microscopic concepts as non-locality or entanglement can be applied in formulating quantum *metaphors* for homeopathy,^{13–17} which are very useful in description¹⁸ of its frontier problems.

The main objective of the present work is introducing, on the basis of the previously obtained results,^{19,20} a new class of quasi-quantum phenomena responsible – among others – for biological growth (regression) and crystallization (dissolution) processes. The proposed model is introduced in the framework of the generalized quantum theory including space-like (non-local) fields. Quasi-quantum phenomena seem to be a missing link between micro- and macro-levels of the matter organization, which provide strong arguments for justifying *ad hoc* formulated macroscopic versions of quantum entanglement^{6–9} and non-locality¹⁰ employed in construction of quantum *metaphors* for homeopathy.^{13–18} In particular the quasi-quantum phenomena are consistent with the weak quantum theory⁶ predicting existence of quantum formalism without Planck's constant and probabilistic interpretation, which can be applied in the macroscopic areas, for instance, in philosophy, psychology and medicine.

The paper is organized as follows. The basic concepts of the growth (decay) according to the first- and second-order Gompertzian kinetics, non-locality and entanglement as

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well as the minimum-uncertainty space-like coherent states of the Morse oscillator are presented in [Appendix](#). They are employed to define the quasi-quantum phenomena in the main part of the article, in which some possible applications of the results obtained in interpretation of the homeopathic remedies activity are proposed and connections with existing theories and discoveries in the field are discussed.

Quasi-quantum phenomena

It is generally assumed that micro- and macro-worlds are characterized by the two classes of phenomena: quantum and classical. The quantum properties of micro-objects (particle, atom, molecule) are coded in the wave equations (11) and (12), whose eigenfunctions define the state of the object, whereas eigenvalues represent so-called observables, for example energy or momentum. The quantum world is characterized, among other, by the following phenomena: quantization of observables – they can take discrete values; superposition and coherence of quantum states; position–momentum and time–energy uncertainty described by the Heisenberg's relations; non-local long-range correlations characterized by the Bell's inequality and entanglement of distinct quantum states and objects *via* e.g. Einstein–Podolski–Rosen channel. On the other hand, the classical level of the matter organization is described by the Hamilton and Lagrange equations of motion for objects moving on the well defined time- and space-trajectories. Their observation does not change the state of the object hence, both time and energy, position and momentum are determinable at the level of experimental accuracy. The macro-observables are not quantized, so the energy and momentum of macro-object can take arbitrary real values and the state of the system is unequivocally determined by the Hamilton or Lagrange functions and the initial conditions.

On the border line between micro- and macro-levels of matter organization, there are the macroscopic quantum phenomena of the first and second kind.^{11,12} Additionally, the results presented in the [Appendix](#) admit existence of the macroscopic quantum phenomena of the third kind appearing in the Gompertzian systems. The notion *Gompertzian system* denotes, for example, living systems (organ, tissue, organism, population of organisms, bacterial colony, tumour) as well as crystals composed of atoms, molecules or ions, whose growth (decay) is described by the Gompertz function (1). The Gompertz growth function belongs to the wide class of sigmoidal (S-shaped) functions and describes exponential growth, which then is exponentially retarded and saturated as time continues. The Gompertzian growth is a result of two classes of competitive processes: the first process stimulates growth and the second constrains growth at the saturation stage. The modified Gompertz function (3) has also been applied to describe the crystallization (dissolution) process. The process of crystallization consists of two stages: (i) nucleation in which molecules come into contact and interact to form spatially ordered structures, and (ii) crystal growth which is the enlargement of the interacting nuclei. Since there are several analogies between

crystallization and bacterial growth (growth of bacteria resembles nucleation and crystal's enlargement, whereas bacterial consumption of nutrients resembles a decrease in supersaturation), this process can be described by the modified Gompertz function (3). It can be proved that this function satisfies the second-order differential equation (8), representing the so-called second-order Gompertzian kinetics. It is interesting to note, that equation (8) is a special case of the more general quantum space-like Horodecki–Feinberg equation (17) for the time-dependent Morse potential (16). In particular, a comparison of the macroscopic equation (8) for the Zwietering–Gompertz function (7) with the quantum equation (17) and its ground state solution (19) for the time-dependent Morse oscillator, leads to the following conclusions:

1. The second-order macroscopic equation governing the Gompertzian growth is a special case of the quantum Horodecki–Feinberg equation for the time-dependent Morse oscillator.
2. The quantum wave function for the ground state of the Morse oscillator for $x_e = 1$ is reduced to the macroscopic Zwietering–Gompertz function.
3. The transition from the micro- to macro-domain can be done by substitution $x_e = 1$ and $v = 0$ in the quantum formulae.
4. The macroscopic second-order Gompertzian kinetics is described by the quasi-quantum equation whose eigenvalue is quantized and takes only one value $\frac{1}{4}$.
5. The Gompertzian systems are characterized by only one eigenfunction representing the fundamental mode of growth.
6. The eigenvalue $\frac{1}{4}$ is equal to the dimensionless dissociation energy of the Morse oscillator, hence the latter is in the dissociation state – there is no oscillation in time and the evolution of the system is consistent with the arrow of time.

The presented above conclusions are revolutionary as they reveal existence of a new, so far unknown, class of the macroscopic quantum (quasi-quantum) phenomena appearing in the Gompertzian systems. The notion *quasi-quantum* refers to the possibility of application of the quantum language and formalism in description of macroscopic phenomena like biological growth or crystallization process. In particular the second-order Gompertzian kinetic equation (8) takes identical form as the microscopic eigenvalue equation for the quantized state of growth. Since the second-order growth equation (8) is a special case of the non-local Horodecki–Feinberg equation (17), the Gompertzian growth belongs to the class of macroscopic non-local phenomena. The nature of this non-locality can be explained having introduced another quasi-quantum phenomenon – macroscopic coherence. If we put $x_e = 1$ into quantum annihilation and creation equations (25) and (26), the former reduces to the equation describing growth whereas the latter – regression (decay) of a macroscopic Gompertzian system. The results obtained indicate that the quantum equations describing non-local coherent states of time-dependent Morse oscillator with anharmonic constant equal to one have identical form as the

equations describing Gompertzian growth and regression. Since such states evolve coherently in space on the localized time trajectory, spatial coherence appears as immanent property of the Gompertzian growth and decay. Macroscopic spatial coherence and non-locality coupled together explain formation of the specific growth patterns in the Gompertzian systems, which is a result of non-local long-range cooperation between the micro-components of the system and the system as a whole. Micro-components can be identified with cells of the biological system or with atoms, molecules, ions in the crystal. In view of the above, the quasi-quantum model explains spatial, long-range coordination of numerous biological functions and entities leading to integration of the micro- and macro-levels of organization of the living systems. It demonstrates also that both crystallization and dissolution processes can be interpreted in terms of the space-like coherent states of the time-dependent Morse oscillator. Another quasi-quantum effect which appears in the Gompertzian systems is the time–energy uncertainty. It is easy to prove that the macroscopic Zwietering–Gompertz function (7) minimizes^{19,20} the time–energy uncertainty relation (22). This result indicates that the macroscopic growth function (7) behaves like a quantum function, which minimizes the time–energy uncertainty relation. It means that the Heisenberg relation despite of the presence in it the Planck's constant is fulfilled not only in micro- but also in the macro-domain.

Quasi-quantum homeopathy

The quasi-quantum phenomena appearing in the Gompertzian systems justify the application of the quantum formalism and language in description of the macroscopic domain. In particular the crystal growth (dissolution) according to the Zwietering–Gompertz function (7) can be interpreted in terms of non-local coherent states of the

time-dependent Morse oscillator. Such processes are spatially coherent – it means that the formation of the specific growth patterns in the Gompertzian systems is a result of a non-local long-range cooperation between micro-level represented by the micro-components of the system and the macro-level representing the system as a whole. In the case of crystallization, the micro-level is identified e.g. with molecules, whereas the macro-level with the crystal i.e. system of interacting molecules. In this picture the Zwietering–Gompertz growth function (7) describes the growth of the spatially coherent macroscopic ensemble in which a large number of molecules collectively cooperate in the ground coherent mode of growth. If crystallization and its reciprocal process of dissolution belong to the class of quasi-quantum non-local coherent phenomena there exists a direct link to homeopathy: the molecules of the remedy prepared in the process of dilution of the active substance are interconnected at-a-distance by non-local correlations, which can be viewed as quasi-quantum (macroscopic) version of the quantum entanglement (see Figure 1). In this respect, the spatially correlated system of molecules can be described in the framework of a multi-particle counterpart of the famous Einstein–Podolski–Rosen (EPR) thought experiment,^{29,30} in which a measurement of the state of one component of the system gives information on the state of the other entangled components of the same system, separated by an arbitrary distance. According to the Bohr's interpretation of this effect,³⁰ the entangled micro-objects form an inseparable quantum ensemble whose inseparability does not depend on the spatial distance between its components. Adopting this interpretation to the molecules of the active substance one may admit that they form inseparable molecular system despite of spatial separation of constituents, caused by the dilution process. Therefore, each

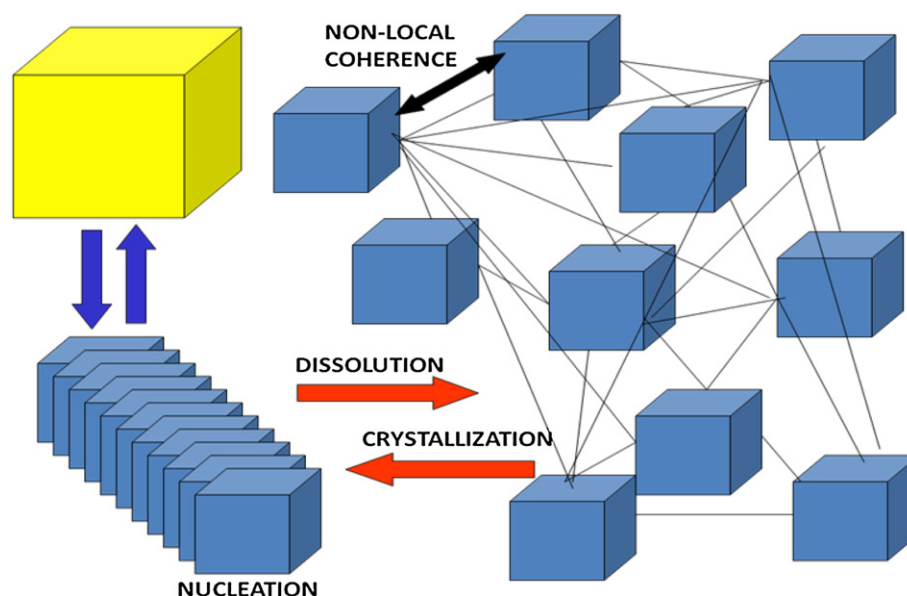


Figure 1 The dissolution and its reciprocal process of crystallization of an active substance as the macroscopic non-local (space-like) coherent phenomenon. The active molecules of the remedy are interconnected at-a-distance by non-local correlations, which can be viewed as quasi-quantum counterpart of the quantum entanglement.

molecule of the active substance is non-locally interconnected at-a-distance with the other active molecules removed off the medicine during its preparation. In such circumstances the therapeutically active ingredient is the molecule carrying information about the state (property, activity) of all molecules of the active substance. This information is administered during therapy. The interpretation proposed has an analog – *holographic effect*, in which the part of the hologram possesses information coded in the whole hologram. The term hologram was introduced by Dennis Gabor in 1947; it is derived from the Greek words ὅλος (*hólos*) (whole) and γραφή (*grafē*) (writing, drawing). Holography is a technique which allows the light scattered from an object to be recorded in the holographic material in such a way that each part of the hologram is able to reproduce the whole image of the object, when illuminated by a laser beam. Holographic materials contain crystals of the active substance around 30 nm or less in diameter. To make such crystals, it is necessary to use conditions which allow crystal formation at the nucleation stage but effectively prevent crystal enlargement. According to Michael Talbot³¹ interpretation, the *whole in every part* nature of a hologram provides us with an entirely new way of understanding organization and order. If we try to take apart something constructed holographically, we will not get the pieces of which it is made, we will only get smaller wholes. In the holographic model of the homeopathic activity, the remedy should contain at least one molecule of the active substance. In the case of the dilutions beyond Avogadro number, when no molecules of the active substance are present in remedy, one may indicate a similar phenomenon for the molecules of the solvent (water, ethanol, lactose) used in producing homeopathic remedies. When the active substance is successively diluted in the solvent and vigorously shaken with striking against an elastic surface in a successive procedure two processes on the micro-level take place: first – molecular dispersion of the conglomerates of the substance, and second – removal of the active molecules of the tincture in the series of dilutions. Both processes result in diminishing the mean distance between molecules of the solvent and increasing electrostatic interactions between them, which are conditions *sine qua non* for appearance of nucleation – prerequisite to formation of ordered quasi-crystal structures of the solvent. Because crystallization can be viewed as non-local coherent phenomenon, the dilution generates molecules of the solvent in the correlated (quasi-entangled) state amenable to form complex structures against decoherence due to collisions with other molecules, exchanging the electromagnetic radiation and chaotic thermal influences. The results reported by Del Giudice *et al*²³ confirmed that the water molecules can move in highly correlated and ordered way due to interactions between water electric dipole and radiation field, which produce ordered quasi-crystal structures in macroscopic domain, i.e. within a few hundred microns. According to Weingärtner,^{4,5} such correlated molecular configurations can be effective carriers of information between molecules of the active substance and molecules of the solvent, during preparation of homeopathic medicines.

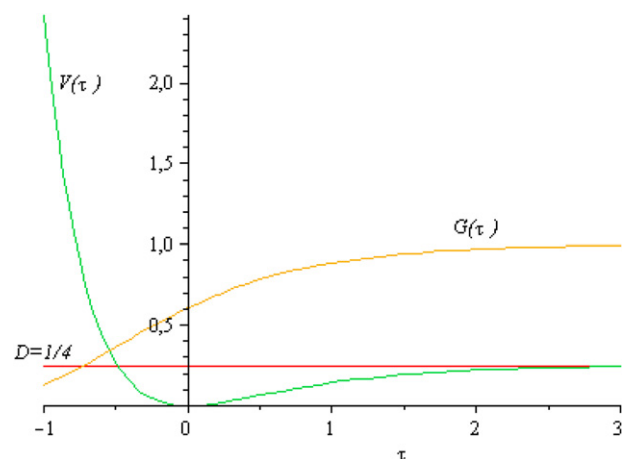


Figure 2 The plots of the dimensionless Gompertz growth function $G(\tau)$, Morse potential $V(\tau)$ and dissociation level $D = 1/4$.

Hence, they are reliable candidates for material carriers of information that is administered during a homeopathic treatment. This interpretation admits homeopathic activity of the remedy even in the case when no molecules of the active substance are present in it. A model, quantitatively describing this phenomenon, is under construction.

Conclusions

It is generally assumed that entanglement and co-existing coherence and non-local correlations appear only in the microscopic quantum domain. Hence, they are of not much interest for macroscopic everyday life as entanglement is completely broken up at the macroscopic level. It is argued that it is mainly due to decoherence – one of the crucial reasons why macroscopic objects never show quantum behaviour. They are so big that they interact strongly with the environment and lose their coherence and entanglement very quickly. In other words, the absence of quantum behaviour in the macro-domain arises naturally due to the increasing difficulty of isolating objects of increasing size and complexity from their environment. The interactions of the micro-objects with the environment result in information transfer that is responsible for the ‘classical’ behaviour of quantum objects. However, according to other opinions,^{3–10,13–18} entanglement, non-locality and coherence might have played a major role in such areas as philosophy, psychology, homeopathy and alternative healing. In particular the macroscopic non-locality and entanglement may play a role in the hypothetical macroscopic quantum phenomena including instantaneous, acausal correlations and influences between patient, practitioner and remedy,^{13–17} ultimately leading to the observed improvement in the patient’s state of health. Additionally macroscopic non-locality has the advantage of being applicable outside homeopathy to other healing disciplines, especially those referring to holistic concepts of biological reality as, for example, the Jung’s synchronicity.¹⁰

The results obtained in this work, particularly the possibility of appearance of the coherence, non-locality, quantization, uncertainty in the macro-world, are in contradistinction to the orthodox claim that the quantum

theory and its implications apply only within the domain of micro-particle physics. Now, it is clear that the strangest outcomes of quantum theory – non-locality, coherence, quantization, uncertainty – are not size-limited and may play important role in the living systems, crystal formation and homeopathy. In particular, it has been proved²⁰ that macroscopic function (7) minimizes the time–energy uncertainty relation (15) including the Planck’s constant. This result reveals that the quantum formalism can be directly applied to the macroscopic function describing biological or substantial growth.

It is well-known that entanglement occurs when parts of a system are non-locally correlated at-a-distance in such a way that measurement of one part of the system instantaneously (i.e. not limited by the speed of light as the upper limit) provides information about its other parts, regardless of their spatial separation. Such a non-local communication channel enables each micro-component to obtain information about the state of the system as a whole and respond to it adequately. This response requires self-organization of the system and effective cooperation of all its interconnected sub-elements, whereas the non-local character of this connectedness enables the system to form coherent complex patterns under the influence of external and internal conditions. In view of this, the non-local spatial coherence of the biological and substantial growth according to the Gompertz function (1) explains why this function almost perfectly describes the growth of organisms, organs, tumours, bacterial colonies, demographic systems and crystals.

The coherent growth of the Gompertzian systems according to the second-order kinetics is driven by the time-dependent counterpart of the Morse potential and resembles the dissociation state of the anharmonic oscillator. In this respect, the model proposed is consistent with the Hankey²¹ concept of the macroscopic coherence at critical instabilities and the Fröhlich model²² of the macroscopic quantum coherence in the biological systems treated as the coupled oscillators in a heat bath supplied with energy at a constant rate – when it exceeds a certain mean rate then oscillators condense into one giant dipole whose sub-elements are spatially interconnected to each other.

The macroscopic second-order Gompertzian kinetics is governed by the quasi-quantum equation including quantized eigenvalue. This equation does not contain the Planck’s constant and its eigenfunction is not interpreted in probabilistic terms. Hence, it is consistent with the predictions of the weak quantum theory developed by Atmanspacher and coworkers.⁶ Also creation–annihilation equations for the coherent states of the Morse oscillator for anharmonic constant equal to one are deprived Planck’s constant and have identical form as the macroscopic equations of the Gompertzian growth and decay.

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I am indebted to Dr Ewa Czerwinska, the head of the Polish Homeopathy Association, for inspiration and stimulation to write this work.

Appendix

Completion of characterisation of the quasi-quantum phenomena requires introduction of some indispensable physical concepts, mathematical formulae and transformations. They are collected in the subsections presenting the first- and second-order Gompertzian kinetics, the concept of non-locality and entanglement as well as the minimum-uncertainty space-like coherent states of the time-dependent Morse oscillator.

Gompertzian systems

In 1825 Benjamin Gompertz²⁴ introduced in the framework of actuarial mathematics the sigmoidal (S-shaped – see Figure 2) function

$$G(t) = G_0 \exp\left\{\frac{b}{a}[1 - \exp(-at)]\right\}, \quad (1)$$

which permits calculating the force of mortality. Nowadays it is employed in biology and medicine to describe the growth of living systems (organ, tissue, organism, population of organisms, bacterial colony, tumour). Here G_0 stands for the initial mass, volume, radius or number of cells or entities, whereas the constants a and b ($a, b > 0$) characterize the dynamics of growth (regression) according to the first-order Gompertzian kinetics

$$\frac{d}{dt}G(t) = b \exp(-at)G(t), \quad \frac{d}{dt}G(t)^\dagger = -b \exp(-at)G(t)^\dagger. \quad (2)$$

Here $G(t)^\dagger = G(t, b \rightarrow -b)$ defines the Gompertzian system in the regression (decay) phase. The modified Gompertz function

$$Z(t) = Z_0 \exp[\pm \exp(-at + d)], \quad a = \frac{\mu e}{Z_0}, \quad d = \frac{\mu e \lambda}{Z_0} + 1 \quad (3)$$

introduced by Zwietering *et al*²⁵ can also be applied to describe the crystallization process,^{26,27} in particular the crystal growth (–) and reverse process of crystal decay (+) (dissolution). In the case of growth, Z_0 stands for the maximal value reached by $Z(t)$ as $t \rightarrow \infty$, μ is the maximum specific growth rate defined as the tangent in the inflection point, λ denotes the lag time defined as the t -axis intercept of that tangent whereas $e = 2.718281\dots$ is the Neper’s number. The Zwietering–Gompertz growth and decay functions (3) are the solutions of the first-order differential equations

$$\begin{aligned} \frac{d}{dt}Z(t) &= a \exp(-at + d)Z(t), \\ \frac{d}{dt}Z(t)^\dagger &= -a \exp(-at + d)Z(t)^\dagger \end{aligned} \quad (4)$$

in which $Z(t)^\dagger$ denotes decay (dissolution) state. Employing the dimensionless variable

$$\tau = \frac{a(t - t_e)}{\sqrt{2}}, \quad t_e = \frac{1}{a}[\ln(2) + d], \quad (5)$$

the equations of growth and decay (4) can be transformed to the form

$$\frac{1}{\sqrt{2}} \left[\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2}\tau)}{\sqrt{2}} - \sqrt{\frac{1}{2}} \right] Z(t) = 0,$$

$$\frac{1}{\sqrt{2}} \left[-\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2}\tau)}{\sqrt{2}} - \sqrt{\frac{1}{2}} \right] Z(t)^\dagger = 0 \quad (6)$$

in which

$$Z(\tau) = Z_0 \exp \left[-\frac{1}{2} \exp(-\sqrt{2}\tau) \right],$$

$$Z(\tau)^\dagger = Z_0^\dagger \exp \left[+\frac{1}{2} \exp(-\sqrt{2}\tau) \right]. \quad (7)$$

By differentiating $Z(\tau)$ twice with respect to τ -coordinate one may prove that this function satisfies also the equation²⁰

$$-\frac{1}{2} \frac{d^2 Z(\tau)}{d\tau^2} + \frac{1}{4} \{ 1 - \exp[-\sqrt{2}\tau] \}^2 Z(\tau) = \frac{1}{4} Z(\tau) \quad (8)$$

representing the so-called second-order Gompertzian kinetics in $Z(\tau)$ representation. It is easy to demonstrate that the above equation is also satisfied²⁰ by the original Gompertz function of growth (1).

$$G(\tau) = G_\infty \exp \left[-\frac{1}{2} \exp(-\sqrt{2}\tau) \right], \quad G_\infty = G_0 \exp \left(\frac{b}{a} \right) \quad (9)$$

expressed in the dimensionless variable

$$\tau = \frac{a(t - t_e)}{\sqrt{2}}, \quad t_e = -\frac{1}{a} \ln \left(\frac{a}{2b} \right), \quad (10)$$

which provides the second-order Gompertzian kinetics in the $G(\tau)$ representation.

Non-locality and entanglement

The local (time-like) systems move with subluminal velocities and interactions between them are restricted – according to Einstein’s special relativity – by the velocity of light in vacuum. In the case of non-local (space-like) systems their motion and interactions or mutual influences (e.g. correlations) between constituents are not subjected to limitation involving the velocity of light as the upper limit. In such systems the interactions at-a-distance including instantaneous correlations between objects separated even on macroscopic distances are possible.²⁸ Since the pioneer work by Einstein, Podolsky and Rosen (EPR) was published^{29,30} the non-local systems have become the subject of intensive studies. The experiments by Aspect and his coworkers^{32,33} proved that separable quantum systems can be interconnected *via* non-local EPR correlations. According to Schrödinger³⁴ a source of this connectedness is *entanglement* defined in the following manner: “When two systems, whose states are known by their respective representatives, enter into temporary physical interaction due to

known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before... By the interaction the two representatives have become entangled”. Quantum entanglement is a physical resource associated with the non-classical correlations that are possible between separated elements of quantum systems. Entanglement can be measured, transformed, purified^{35,36} and employed in quantum communication and computing,^{37,38} superdense coding and cryptography,^{35,36} as well as in teleportation of quantum states.^{37,38}

Despite of non-local interconnectedness of quantum objects, the EPR effect and entanglement cannot be used to send superluminal signals from one object to another – there is no means to transfer messages *via* this procedure.^{39–42} However, this prohibition is broken if quantum mechanics is allowed to be ‘slightly’ non-linear⁴³ or is interpreted in transactional⁴⁴ or tachyonic⁴⁵ terms. In view of this we cannot exclude a possibility of existence of a space-like transfer of information between micro-objects even though no material signal or energy is passed between them.⁴⁶

In the local quantum systems the fundamental role plays the Schrödinger equation describing a particle of mass m moving with subluminal velocity in the field of the scalar potential $V(r)$

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dr^2} + V(r)\psi = E\psi. \quad (11)$$

In the generalized non-local quantum theory a particle of mass m moving with superluminal velocity in the field of the time-dependent vector potential $V(t)$ is described by the Horodecki–Feinberg equation^{47,48}

$$-\frac{\hbar^2}{2mc^3} \frac{d^2 \Psi}{dt^2} + \frac{1}{c} V(t)\Psi = P\Psi. \quad (12)$$

Here ψ denotes the wave function associated with a particle of energy E , $\hbar = 105,457,266 \times 10^{-34}$ J s is the Planck constant divided by 2π , c is the light velocity, whereas Ψ represents a non-local matter wave associated with the superluminal particle of momentum P . Equation (12) represents the non-relativistic version of the relativistic Feinberg equation⁴⁷ for non-local faster than light objects. It has been derived by Horodecki⁴⁸ by taking advantage the same procedure as that used in deriving the Schrödinger equation (11) from the relativistic Klein–Gordon equation for local slower than light particles.⁴⁸

Time- and space-like coherent states

The minimum-uncertainty coherent states $|\alpha\rangle$ are defined as eigenstates of the annihilation operator⁴⁹

$$\widehat{A} |\alpha\rangle = \alpha |\alpha\rangle. \quad (13)$$

They minimize the Heisenberg position–momentum uncertainty relation

$$\Delta r^2 \Delta p^2 \geq -\frac{1}{4} \langle \alpha | [r, \widehat{p}] | \alpha \rangle^2 = \frac{\hbar^2}{4} \quad (14)$$

in which $\hat{p} = -\hbar d/dr$ denotes the momentum operator, whereas $[r, \hat{p}] = r\hat{p} - \hat{p}r = \hbar$ is the commutator of the position–momentum operators. This definition is useful in construction of the coherent states of anharmonic oscillators describing, for example, vibrational motion of molecular systems. The coherent states defined above are local (time-like), it means that they are associated with subluminal objects and propagate coherently in time on the well defined space trajectory.⁵⁰ If we take into account the Horodecki–Feinberg equation one may introduce the non-local (space-like) coherent states of anharmonic oscillators. Those states are associated with superluminal objects and propagate coherently in space along the well defined time trajectory.²⁰ Such states minimize the time–energy uncertainty relation²⁰

$$\Delta t^2 \Delta E^2 \geq -\frac{1}{4} \langle \alpha | [t, \hat{E}] | \alpha \rangle^2 = \frac{\hbar^2}{4} \quad (15)$$

in which $\hat{E} = \hbar d/dt$ denotes the energy operator.

Space-like coherent states of the Morse oscillator

Employing the time-dependent version of the Morse potential⁵¹

$$V(t) = D_e \left\{ 1 - \exp[-a(t - t_e)] \right\}^2 \quad (16)$$

one may construct²⁰ the minimum-uncertainty coherent states of the time-dependent Morse oscillator. Here, D_e is the potential depth identified with the dissociation energy of the oscillator, a is range parameter, whereas t_e is equilibrium time in which potential (16) attains minimum equal to zero. The Morse oscillator belongs to the wide class of anharmonic vibrating systems, whose components (e.g. atoms) move under the effect of the force generated by the potential (16). Each anharmonic potential should satisfy the two fundamental conditions: $V(t \rightarrow \infty) = D_e$ (dissociation state) and $V(t = t_e) = 0$ (potential minimum).

In order to construct the space-like coherent states of the Morse oscillator, which minimizes the time–energy uncertainty relation (15), we employ the Horodecki–Feinberg equation (12) including potential (16)

$$\left\{ -\frac{1}{2} \frac{d^2}{d\tau^2} + \frac{1}{4x_e} \left[1 - \exp(-\sqrt{2x_e}\tau) \right]^2 - \left(v + \frac{1}{2} \right) + x_e \left(v + \frac{1}{2} \right)^2 \right\} \Psi_v = 0 \quad (17)$$

expressed in the dimensionless coordinate

$$\tau = \frac{a(t - t_e)}{\sqrt{2x_e}}, \quad x_e = \frac{\hbar a}{c\sqrt{8mD_e}} \quad (18)$$

in which x_e stands for the anharmonic parameter characterizing the oscillator. The eigenvalues of this equation are quantized, it means that they take discrete values according to the quantum number $v = 0, 1, 2, \dots$, whereas the wave function of the ground states ($v = 0$) reads

$$\Psi_0 = N_0 \exp \left[-\frac{1}{2x_e} \exp(-\sqrt{2x_e}\tau) \right] \exp \left[-\frac{1}{\sqrt{2x_e}} (1 - x_e)\tau \right]. \quad (19)$$

The Horodecki–Feinberg equation can be factorized using the creation and annihilation operators²⁰

$$\begin{aligned} \hat{A}^\dagger \hat{A} \Psi_v &= E_v - E_0 \Rightarrow \hat{A}^\dagger \hat{A} \Psi_0 = 0, \\ \hat{A}^\dagger &= \frac{1}{\sqrt{2}} \left[-\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2x_e}\tau)}{\sqrt{2x_e}} - \sqrt{\frac{x_e}{2}} \right], \\ \hat{A} &= \frac{1}{\sqrt{2}} \left[\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2x_e}\tau)}{\sqrt{2x_e}} - \sqrt{\frac{x_e}{2}} \right]. \end{aligned} \quad (20)$$

The non-local coherent states of the time-dependent Morse oscillator are eigenstates of the annihilation operator²⁰

$$\begin{aligned} \hat{A} |\alpha\rangle &= \alpha |\alpha\rangle, \quad |\alpha\rangle = \exp \left[-\frac{1}{2x_e} \exp(-\sqrt{2x_e}\tau) \right] \\ &\exp \left[-\frac{1}{\sqrt{2x_e}} (1 - x_e)\tau \right] \exp \left[\alpha \sqrt{2}\tau \right]. \end{aligned} \quad (21)$$

One may prove^{19,20} that they minimize the time–energy uncertainty relation expressed in the Morse variable $T(\tau)$ ³⁵

$$\begin{aligned} \Delta T(\tau)^2 \Delta E^2 &= \frac{\hbar^2}{4} \langle \alpha | \exp(-\sqrt{2x_e}\tau) | \alpha \rangle^2, \\ T(\tau) &= \frac{1 - \exp(-\sqrt{2x_e}\tau)}{\sqrt{2x_e}} - \sqrt{\frac{x_e}{2}}. \end{aligned} \quad (22)$$

Micro–macro correspondences

Comparison of the macroscopic second-order differential equation (8) for the Zwietering–Gompertz function (7), with the quantum Horodecki–Feinberg equation (17) and its ground state solution (19) for the time-dependent Morse oscillator, reveals the following relationships

$$\begin{aligned} \left\{ -\frac{1}{2} \frac{d^2}{d\tau^2} + \frac{1}{4x_e} \left[1 - \exp(-\sqrt{2x_e}\tau) \right]^2 - \left(v + \frac{1}{2} \right) + x_e \left(v + \frac{1}{2} \right)^2 \right\} \Psi_v \xrightarrow{x_e=1, v=0} &\left\{ -\frac{1}{2} \frac{d^2}{d\tau^2} \right. \\ &\left. + \frac{1}{4} \left[1 - \exp(-\sqrt{2}\tau) \right]^2 - \frac{1}{4} \right\} Z(\tau) = 0. \end{aligned} \quad (23)$$

$$\begin{aligned} \Psi(\tau)_v \xrightarrow{x_e=1, v=0} &Z(\tau) \\ N_0 \exp \left[-\frac{1}{2x_e} \exp(-\sqrt{2x_e}\tau) \right] \exp \left[-\frac{1}{\sqrt{2x_e}} (1 - x_e)\tau \right] \xrightarrow{x_e=1} &Z_\infty \exp \left[-\frac{1}{2} \exp(-\sqrt{2}\tau) \right]. \end{aligned} \quad (24)$$

They evidently prove that the second-order Gompertz equation of growth (8) is a special case of the more general quantum non-local Feinberg–Horodecki equation (17), whose ground state eigenfunction ($v=0$) for $x_e=1$ is reduced to the Zwietering–Gompertz growth function (7).

Similar correspondences can be obtained for the macroscopic first-order kinetics (6) describing crystal growth and decay. Introducing $x_e=1$ into the quantum annihilation–creation equations (20) one gets the macroscopic equations of the crystal growth and regression

$$\hat{A}|0\rangle = \frac{1}{\sqrt{2}} \left[\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2x_e}\tau)}{\sqrt{2x_e}} - \sqrt{\frac{x_e}{2}} \right] |0\rangle \xrightarrow{x_e=1} \frac{1}{\sqrt{2}} \left[\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2}\tau)}{\sqrt{2}} - \sqrt{\frac{1}{2}} \right] |0\rangle \xrightarrow{\tau \rightarrow t} \frac{dZ(t)}{dt} - a \exp(-at + d)Z(t) = 0. \quad (25)$$

$$\langle 0|\hat{A}^\dagger = \langle 0| \frac{1}{\sqrt{2}} \left[-\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2x_e}\tau)}{\sqrt{2x_e}} - \sqrt{\frac{x_e}{2}} \right] \xrightarrow{x_e=1} \langle 0| \frac{1}{\sqrt{2}} \left[-\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2}\tau)}{\sqrt{2}} - \sqrt{\frac{1}{2}} \right] \xrightarrow{\tau \rightarrow t} \frac{dZ(t)^\dagger}{dt} + a \exp(-at + d)Z(t)^\dagger = 0. \quad (26)$$

Here, $|0\rangle$ and $\langle 0|$ denote the ground state eigenfunctions of the annihilation and creation operators. It can be proved that the macroscopic Zwietering–Gompertz function minimizes the time–energy uncertainty relation^{11,12}

$$\Delta T(\tau)^2 \Delta E^2 \geq -\frac{1}{4} \langle 0| [T(\tau), \hat{E}] |0\rangle^2, \quad [T(\tau), \hat{E}] = i\hbar \exp(-\sqrt{2}\tau) \quad (27)$$

in which $T(\tau)$ is the Morse variable for $x_e=1$ whereas $|0\rangle$ denotes the Zwietering–Gompertz function of growth (7)

$$T(\tau) = -\frac{1}{\sqrt{2}} \exp(-\sqrt{2}\tau), \quad \hat{E} = i\hbar \frac{d}{d\tau}, \quad \Delta T^2 = \langle 0| T(\tau)^2 |0\rangle - \langle 0| T(\tau) |0\rangle^2, \quad \Delta E^2 = \langle 0| \hat{E}^2 |0\rangle - \langle 0| \hat{E} |0\rangle^2, \quad \Delta T^2 \Delta E^2 = \frac{\hbar^2}{4} \langle 0| \exp(-\sqrt{2}\tau) |0\rangle^2. \quad (28)$$

This result clearly indicates that the quantum formalism can be directly applied to the macroscopic functions describing crystal growth. It also proves that the Zwietering–Gompertz growth function represents the so-called *intelligent* coherent states; they not only minimize the time–energy uncertainty

relation but also maintain this relation in space due to its spatial stability. Such non-local states differ from the ordinary time-like coherent states, which maintain the position–momentum Heisenberg relation in time due to its temporal stability.⁵²

References

- Vittorio E, Marcella N. Thermodynamics of extremely diluted aqueous solutions. *Ann N Y Acad Sci* 1999; **827**: 241–248.
- Shepperd J. Chaos theory: implications for homeopathy. *J Am Inst Homeopath* 1994; **87**: 22–29.
- Weingärtner O. Homeopathy and quantum field theory. *Forsch Komplementarmed* 2006; **13**: 140.
- Weingärtner O. The homeopathic mechanism from the viewpoint of a quantum mechanical paradoxon. *J Altern Complement Med* 2005; **11**: 773–774.
- Weingärtner O. What is the therapeutically active ingredient of homeopathic potencies? *Homeopathy* 2003; **92**: 145–151.
- Atmanspacher H, Romer H, Walach H. Weak quantum theory: complementarity and entanglement in physics and beyond. *Found Phys* 2002; **32**: 379–406.
- Walach H. Entanglement model of homeopathy as an example of generalized entanglement predicted by weak quantum theory. *Forsch Komplementarmed* 2003; **10**: 192–200.
- Hyland ME. Does a form of “entanglement” between people explain healing? An examination of hypotheses and methodology. *Complement Ther Med* 2004; **12**: 198–208.
- Hyland ME. Entanglement and some heretical thoughts about homeopathy. *Homeopathy* 2004; **94**: 105–106.
- Walach H. Magic of signs: a non-local interpretation of homeopathy. *Br Homeopath J* 2000; **89**: 127–140.
- Takagi S. *Macroscopic quantum tunnelling*. Cambridge: University Press, 2002.
- Leggett AJ. *The problems of physics*. Oxford: University Press, 1987.
- Milgrom LR. Patient–practitioner–remedy entanglement. Part 1: A qualitative, non-local metaphor for homeopathy based on quantum theory. *Homeopathy* 2002; **91**: 239–248.
- Milgrom LR. Patient–practitioner–remedy entanglement. Part 2: Extending the metaphor for homeopathy using molecular quantum theory. *Homeopathy* 2003; **92**: 35–43.
- Milgrom LR. Patient–practitioner–remedy entanglement. Part 3: Refining the quantum metaphor for homeopathy. *Homeopathy* 2003; **92**: 152–160.
- Milgrom LR. Patient–practitioner–remedy entanglement. Part 4: Towards classification and unification of the different entanglement models for homeopathy. *Homeopathy* 2004; **93**: 34–42.
- Milgrom LR. Patient–practitioner–remedy entanglement. Part 5: Can homeopathic remedy reactions be outcomes of patient–practitioner–remedy entanglement? *Homeopathy* 2004; **93**: 94–98.
- Milgrom LR. Journeys in the country of the blind: entanglement theory and the effects of blinding on trials of homeopathy and homeopathic proving. *Evid Based Complement Alternat Med* 2007; **4**: 7–16.
- Molski M, Konarski J. Coherent states of Gompertzian growth. *Phys Rev E* 2003; **68**: 021916.
- Molski M. Space-like coherent states of time-dependent Morse oscillator. *Eur Phys J D* 2006; **40**: 411–418.
- Hankey A. Macroscopic quantum coherence in patient–practitioner–remedy entanglement: the quantized fluctuation field perspective. *Evidence-based Complement Altern Med* 2008; **5**: 1–3.
- Fröhlich H. Long range coherence and energy storage in biological systems. *Int J Quantum Chem* 1968; **2**: 641–649.
- Del Giudice E, Preparata G, Vitiello G. Water as a free electron dipole laser. *Phys Rev Lett* 1988; **61**: 1085–1088.

- 24 Gompertz B. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. *Philos Trans R Soc Lond* 1825; **115**: 513–585.
- 25 Zwietering MH, Jongenburger I, Rombouts FM, van't Riet K. Modeling of the bacterial growth curve. *Appl Environ Microbiol* 1990; **56**: 1875–1881.
- 26 Kloek W, Walstra P, Van Vliet T. Crystallization kinetics of fully hydrogenated palm oil in sunflower oil mixtures. *J Am Oil Chem Soc* 2000; **77**: 389–398.
- 27 Foubert I, Vanrolleghem PA, Vanhoutte B, Dewettinck K. Dynamic mathematical model of the crystallization kinetics of fats. *Food Res Int* 2002; **35**: 945–956.
- 28 Tittel W, Brendel J, Gissin B, Herzog T, Zbinden H, Gisin N. Experimental demonstration of quantum correlations over more than 10 km. *Phys Rev A* 1988; **57**: 3229–3232.
- 29 Einstein A, Podolsky B, Rosen N. Can quantum-mechanical description of physical reality be considered complete? *Phys Rev* 1935; **47**: 777–780.
- 30 Bohr N. Can quantum-mechanical description of physical reality be considered complete? *Phys Rev* 1935; **48**: 696–702.
- 31 Talbot M. *Holographic Universe*. Harper Collins, 1991.
- 32 Aspect A, Grangier P, Roger G. Experimental realization of Einstein–Podolsky–Rosen gedanken experiment: a new violation of Bell's inequalities. *Phys Rev Lett* 1982; **48**: 91–94.
- 33 Aspect A, Dalibard J, Roger G. Experimental test of Bell's inequalities using time-varying analyzers. *Phys Rev Lett* 1982; **49**: 1804–1807.
- 34 Schrödinger E. Discussion of probability relations between separated systems. *Proc Camb Phil Soc* 1935; **31**: 555–563.
- 35 Horodecki M, Horodecki P, Horodecki R. Mixed-state entanglement and distillation: Is there a “bound” entanglement in nature? *Phys Rev Lett* 1998; **80**: 5239–5242.
- 36 Horodecki K, Horodecki M, Horodecki P, Oppenheim J. Secure key from bound entanglement. *Phys Rev Lett* 2005; **94**: 160502.
- 37 Lo H-K, Popescu S, Spiller T. *Introduction to quantum computation and information*. Singapore: World Scientific, 1998.
- 38 Bouwmeester D, Pan J-W, Mattle K, Eibl M, Weinfurter H, Zeilinger A. Experimental quantum teleportation. *Nature* 1997; **390**: 575–579.
- 39 Eberhard PH. Bell's theorem without hidden variables. *Il Nuovo Cimento B* 1977; **38**: 75–80.
- 40 Eberhard PH. Bell's theorem and the different concepts of locality. *Il Nuovo Cimento B* 1977; **46**: 392–419.
- 41 Ghirardi GC, Rimini A, Weber T. A general argument against superluminal transmission the quantum mechanical measurement process. *Letters Nuovo Cimento* 1980; **27**: 293–298.
- 42 Ghirardi GC, Grassi R, Pearle P. Comment on explicit collapse and superluminal signals. *Phys Lett* 1992; **166**: 435–438.
- 43 Polchinski J. Weinberg's nonlinear quantum mechanics and the Einstein–Podolsky–Rosen paradox. *Phys Rev Lett* 1991; **28**: 397–400.
- 44 Cramer JG. Quantum nonlocality and the possibility of superluminal effects. In: *Proceedings of the NASA Breakthrough Propulsion*. Cleveland: Physics Workshop, 1977.
- 45 Molski M. The dual de Broglie wave. *Adv Imaging Electron Phys* 1998; **101**: 144–239.
- 46 Stapp H. Are superluminal connections necessary? *Il Nuovo Cimento B* 1977; **40**: 191–204.
- 47 Feinberg G. Possibility of faster-than-light particles. *Phys Rev* 1967; **159**: 1089–1105.
- 48 Horodecki R. Extended wave description of a massive spin-0 particles. *Il Nuovo Cimento B* 1988; **102**: 27–32.
- 49 Zhang WM, Feng DH, Gilmore R. Coherent states: theory and some applications. *Rev Mod Phys* 1990; **62**: 867–927.
- 50 Cooper IL. A simple algebraic approach to coherent states for the Morse oscillator. *J Phys A Math Gen* 1992; **25**: 1671–1683.
- 51 Morse PM. Diatomic molecules according to the wave mechanics. II. Vibrational levels. *Phys Rev* 1929; **34**: 57–64.
- 52 Aragone C, Guerri G, Salamo S, Tani JL. Intelligent spin states. *J Phys A Math Nucl Gen* 1974; **7**: L149–L151.